3 8006 10058 1027

Report No. 59

May, 1952.

THE COLLEGE OF AERONAUTICS

CRANFIELD

Surface Conduction of the Heat Transferred from a Boundary Layer

- by -

T. Nonweiler, B.Sc.

COLLEGE OF AERONAUTICS LIBRARY

---oOo---

SUMMARY

This note considers the effects of thermal conductivity upon the temperature distribution in the skin of a body (moving through the air) due to the heat transferred from the boundary layer. It is found that the effects are of importance only very near the nose of the body, and that here the temperature reaches a maximum which, depending on the skin conductivity and thickness, may be appreciably less than the thermometer temperature, particularly at high speeds and altitudes of flight.

- - - - - - -

YSB

LIST OF CONTENTS

N O T A T I O N

L	representative length along the body surface.
M	$= U/a$, the local Mach Number.
Q	$= k_H \frac{1}{2} \rho_1 U_1^3 \sqrt{x}$.
R_x	$= U_1 x/\nu_1$, the local Reynolds Number.
T	temperature.
T_m	defined in (3.3).
$T_{max.}$	maximum temperature of skin.
T_{th}	'thermometer' temperature.
U	velocity of air relative to body.
a	speed of sound.
c	specific heat of skin.
c_f	local skin friction coefficient (= skin friction per unit area $\div \frac{1}{2} \rho_1 U_1^2$).
d	thickness of skin.
$f()$	$= k_H \sqrt{R_x}$.
k	thermal conductivity of skin. $\frac{[M]}{[X][T]}$ $\frac{ML}{t^3T}$ gm cal ft^{-2} sec^{-1}
k_H	heat transfer coefficient (= heat transfer from boundary layer per unit area per unit time $\div \frac{1}{2} \rho_1 U_1^3$).
$(k_H)_m$	value of k_H at $x = L$.
q	heat flux across skin (in direction of y-increasing).
s	Stefan's constant.
t	time.
x, y	system of cartesian coordinates parallel and perpendicular to body skin, with origin at outer surface of the skin at the nose.
γ	ratio of the specific heats of air.
δ	defined in (3.3).
ε	emissivity.
η	$= \delta^{-8/13} \xi$.
θ	$= T_w/T_m$.
μ	viscosity of air.
ν	$= \mu/\rho$, the kinematic viscosity.

Notation (contd.)

ξ $= x/L$.

ρ air density.

σ Prandtl Number of the air.

τ $= \delta^{1/13}\,\theta$.

ω $= \frac{T}{\mu}\frac{d\mu}{dT}$, assumed to be a constant.

Suffix 0 refers to conditions at 0^{o}C and 1 atm. pressure.

1 refers to conditions just outside the boundary layer.

a refers to conditions in the ambient air at the altitude of flight.

B refers to conditions at the inside surface of the body skin.

W refers to conditions at the outside surface of the body skin.

1. Introduction

An estimate can be made under certain circumstances of the heat transferred from the boundary layer over a moving body to its surface, but the equations governing the temperature distribution within the body are so complicated that some approximation in their solution is necessary. The most commonly adopted assumption is that the conduction of heat along the surface may be ignored, although as far as the transfer of heat across the skin into the body is concerned, it is assumed to be a perfect conductor (i.e. no temperature gradient exists), so that by relating the heat transferred to the body with its thermal capacity an estimate may be made of the rate of increase of body temperature and its final (steady state) value found.

In this note we shall attempt to justify this procedure. We find, in fact, that it is only very near the nose of the body that conductivity is at all important in relation to the longitudinal variation of temperature, and this leads us to an estimate of the maximum temperature reached on the surface of a body which is different from the so-called 'thermometer temperature' reached on a non-conducting skin, the difference being appreciable particularly at high velocities or high altitudes of flight. We shall consider the problem in relation to a body covered with a thin skin, insulated at its inside surface, which enables us to introduce some simplifying assumptions in reaching our answer.

Of the fundamental assumptions implicit in the following discussion, the most important perhaps is that the air in the boundary layer acts as a homogeneous medium. Near the nose where conductivity is important, the boundary layer will be very thin: so thin, in fact, that the molecular mean free path may be of commensurate size. Since the molecules are only capable of transferring a finite amount of heat energy to the surface, it cannot be expected that the theoretically infinite rate of heat transfer at the nose (where the boundary layer originates) will be realised in practice, so that considerations of inhomogeneity may well set a limit on surface temperatures over and above that suggested here.

2. The Basic Problem

We shall consider a two-dimensional body of such a form that the pressure gradient along the surface may be neglected, in which event we may express the local heat transfer coefficient as a function of the following variables:

$$\sqrt{R_x}\, k_H = f\left(M_1,\ \gamma,\ \sigma,\ \omega,\ \frac{T_w}{T_1}\right), \qquad \ldots\ldots (2.1)$$

the symbols being defined at the beginning of this note, and ω may be regarded as determining the variation of viscosity with temperature.

For the laminar boundary layer Crocco[1] has found that approximately

$$\frac{k_H}{c_f} = \frac{1}{2\sigma^{1/6}}\left(\frac{\frac{T_{th}}{T_1} - \frac{T_w}{T_1}}{\frac{T_{th}}{T_1} - 1}\right) \quad \text{......(2.2)}$$

where

$$T_{th} = T_1(1 + \sigma^{\frac{1}{2}}\frac{\gamma-1}{2}M_1^2)$$

is the 'thermometer temperature', and Young[2] has obtained for the skin friction coefficient

$$\sqrt{R_x}\, c_f = 0.664\left[0.45 + 0.55\frac{T_w}{T_1} + 0.09(\gamma-1)M_1^2\sigma^{\frac{1}{2}}\right]^{-(1-\omega)/2}. \quad \text{.....(2.3)}$$

The rate of heat radiation away from the surface of the body is given as

$$\varepsilon_w s T_w^4$$

where s is Stefan's Constant and ε_w is called the 'surface emissivity' depending upon the quality and colour of the outside body surface, and to a certain extent also upon its temperature (T_W), although we propose to neglect this last effect.

The air will also be radiating heat to the surface. An infinite layer of gas will behave as a black body emittor for those frequencies of radiation which it absorbs. For practically all the constituents of air except water vapour, these emitting bands are so narrow that the atmospheric radiation may be ignored, at least in the stratosphere, where water vapour is virtually absent. At any rate, the radiation is at such a low temperature that it is negligible compared with that from the body surface in most problems of practical importance. Strictly, one must also include radiation from the sun and the earth's surface as well, but these too will be ignored here, although they may certainly be important in some problems.

The boundary layer is, however, a part of the gas which is at high temperature, and the question naturally arises as to its radiation. However, the layer is of such a small extent that its emissivity is in no way comparable with that of a black body, and may be neglected compared with, say, the heat input due to conduction. Such a statement needs justification, but this will not be attempted here.

To sum up, the net heat input to the skin we shall treat as

$$q_W = k_H \tfrac{1}{2}\rho_1 U_1^3 - \varepsilon_W s T_W^4 \qquad \text{area/time} \quad \ldots\ldots(2.4)$$

per unit area, and this will only be valid for $\left(\frac{T_W}{T_1}\right)^4 \gg 1$.

The heat output from the body's inner surface is of course something over which the designer has some control. We shall express it as

$$q_B = +\varepsilon_B s T_B^4 \qquad \ldots\ldots(2.5)$$

(measuring heat flux q in each case in the same direction - <u>into</u> the body), where ε_B must be determined in relation to the particular problem. It is quoted in this form since the inside of the body is generally a poor conductor and the heat loss is usually merely by radiation, so that the variation of ε_B with temperature may be small: however, ε_B will only be the surface emissivity if in fact the body is not absorbing heat from internal sources of radiation as well.

The temperature distribution inside the skin of the body will be governed by an equation

SECTION 3b
VOL II
WEATHERBURN

$$k\nabla^2 T = c\frac{\partial T}{\partial t} \qquad \ldots\ldots(2.6)$$

subject to the boundary conditions that, if we measure y in the direction of the normal to the body surface towards the inside of the body, then

$$\left.\begin{aligned} k\left(\frac{\partial T}{\partial y}\right)_W &= -q_W \\ k\left(\frac{\partial T}{\partial y}\right)_B &= -q_B \end{aligned}\right\} \qquad \ldots\ldots(2.7)$$

and the outer surface temperature of the body is given as T_W. In the same way as we assume a two-dimensional body with negligible pressure gradient, we also assume the body skin to be two-dimensional and plane surfaced, which is the most reasonable interpretation of these conditions in supersonic flow.

3. <u>Evaluation of Maximum Temperature</u>

We shall now make two further assumptions. Firstly, that conditions are steady: this is valid for most flight conditions in that region of the surface where the maximum temperature is reached. For initially, at least, the temperature there is less than that for which the heat transfer from the boundary layer is zero (i.e. $T_W < T_{th}$) so that the heat input becomes infinitely large near the nose (because $k_H \propto 1/\sqrt{x}$), with the result that the skin reaches its final steady (equilibrium) value very rapidly. Then we shall also make the assumption that the temperature of the skin does not differ greatly from that at its outside surface. Stated formally we assume that

$$\left(\frac{T - T_w}{T_w} \right) \ll 1 \qquad \ldots\ldots (3.1)$$

But

$$\frac{T - T_w}{T_w} = O\left(\frac{d}{T_w} \frac{\partial T}{\partial y} \right) = O\left(\frac{q_w d}{k T_w} \right) = O\left(\frac{s\, T_{max}^3\, d}{k} \right)$$

since, as we shall see, $q_w = O(s\, T_{max}^4)$ in the region where the wall temperature reaches its maximum value, T_{max}. Putting as typical values: $k = 0.1$ cal/cm. sec. deg. C., $d = 1$ mm, and $T_{max} \leqq 1500^\circ$ K, we find that

$$\frac{s\, T_{max}^3\, d}{k} \leqq \frac{1}{200}$$

so that the skin temperature is within 1 per cent or so of the wall temperature, even where the temperature gradient across the skin is highest.

Integrating equation (2.6) across the width of the skin, i.e. with respect to y from 0 to d, we find that, using the above assumptions:

PG. 269 - 271
ADVANCED MATH
FOR ENGINEERS
REDDICK & MILLER

$$k d \frac{d^2 T_w}{dx^2} + k \left(\frac{\partial T}{\partial y} \right)_B - k \left(\frac{\partial T}{\partial y} \right)_W = 0$$

and from (2.4), (2.5) and (2.7) it follows that

$$\frac{d^2 T_w}{dx^2} = \frac{1}{kd} \left[(\varepsilon_w + \varepsilon_B)\, s\, T_w^4 - k_H \tfrac{1}{2} \rho_1 U_1^3 \right]. \qquad \ldots\ldots (3.2)$$

We now write

$$\xi = x/L, \quad \theta = T_w/T_m, \quad 2\, T_m\, k\, d/L^2 (k_H)_m\, \rho_1 U_1^3 = \delta,$$

$$s\, T_m^4 = \frac{(k_H)_m\, \rho_1 U_1^3}{2(\varepsilon_w + \varepsilon_B)}$$

where

$$(k_H)_m = \frac{1}{\sqrt{R_L}}\, f(M_1, \gamma, \sigma, \omega, \frac{T_m}{T_1}) \qquad \ldots\ldots (3.3)$$

is the heat transfer coefficient at $\xi = 1$, corresponding to a wall temperature T_m; this temperature T_m is that which would be reached at this position in the absence of any conductivity along the skin. Then in (3.2)

$$\delta \frac{d^2\theta}{d\xi^2} = \theta^4 - \frac{k_H}{(k_H)_m}. \qquad \text{......(3.4)}$$

If we now treat $\delta << 1$, then it follows that this has the approximate solution

$$\theta^4 = \frac{k_H}{(k_H)_m} + O(\delta) \qquad \text{......(3.5)}$$

except where

$$\frac{d^2\theta}{d\xi^2} = O(\frac{\theta^4}{\delta}), \qquad \text{......(3.6)}$$

i.e. except where the rate of heat transfer, neglecting surface conductivity, becomes very large. In fact, such a state is reached near the nose where $\xi << 1$. For, on neglecting δ, since from (2.1)

$$\frac{k_H}{(k_H)_m} = \frac{f(M_1, \gamma, \sigma, \omega, \theta \frac{T_m}{T_1})}{\xi^{\frac{1}{2}} f(M_1, \gamma, \sigma, \omega, \frac{T_m}{T_1})} \qquad \text{......(3.7)}$$

as $\xi \to 0$, it follows from (3.5) that

$$\frac{f(\theta \frac{T_m}{T_1})}{\theta^4} \sim \xi^{\frac{1}{2}}. \qquad \text{......(3.8)}$$

This implies that the heat transfer coefficient must tend to zero as $\xi \to 0$, which means that $T_w \to T_{th}$ (see e.g. equations (2.2) and (2.3)). In fact, from (3.8) it would follow that

$$\frac{T_{th}}{T_m} - \theta \sim \xi^{\frac{1}{2}}$$

so that

$$\frac{d^2\theta}{d\xi^2} \sim \xi^{-3/2}$$

and becomes infinitely large near the nose. Hence, by (3.6), equation (3.5) is invalid if

$$\xi = O(\delta^{2/3}). \qquad \text{......(3.9)}$$

To recapitulate, for small values of δ, which means either a thin skin (as already assumed) or a low skin conductivity, the temperature distribution may be obtained simply by equating the heat input from the boundary layer with the heat radiated away from the skin, except near the nose where conductivity has an effect upon the temperature which would otherwise approach the thermometer temperature. An analytical solution is hampered at this stage by the unknown or obscure way in which the heat transfer coefficient varies along the surface. The solution of (3.4) entails a knowledge at least of the variation of k_H with T_w and x (i.e. with θ and ξ). However, we have seen in (2.2) and (2.3) that although k_H may be related to T_w, the relation is a complicated one: at best this solution applies only to laminar boundary layers, and there is no corresponding solution for the turbulent layer. For this reason we must at this stage introduce another simplifying assumption: that we neglect the variation of k_H with T_w; we shall, in fact, now write:

$$k_H = \frac{(k_H)_m}{\xi^{\frac{1}{2}}} = \frac{1}{\sqrt{R_x}} f(M_1, \sigma, \gamma, \omega, \frac{T_m}{T_1}) . \qquad \ldots\ldots(3.10)$$

This is plainly unjustified if in fact T_w approaches T_{th}: as we have set out to examine the effects of conductivity which we have seen will in fact make T_w different from T_{th} at the nose, we are evidently seeking a solution where such a condition (that $T_w \to T_{th}$) is, by implication, absent. If, indeed, the results of our work imply a value of the maximum wall temperature commensurate with the thermometer temperature T_{th}, then our assumptions break down and the inference is that in this example conductivity is unimportant. Bearing in mind our concern with heat transfer at high speeds where T_{th} is large, we may even go so far as to say that if our assumptions break down (and T_w is large also) the results are of little practical interest.

Proceeding on the basis of equation (3.10) and writing

$$\theta = \delta^{-1/13}\,\tau, \quad \text{and} \quad \xi = \delta^{8/13}\,\eta$$

we have in (3.4) that

$$\frac{d^2\tau}{d\eta^2} = \tau^4 - \frac{1}{\eta^{\frac{1}{2}}} . \qquad \ldots\ldots(3.11)$$

The boundary conditions to be applied concern the values of

$$\frac{d\tau}{d\eta} = \delta^{9/13}\frac{d\theta}{d\xi} = \delta^{9/13}\left(\frac{L}{T_m}\frac{dT_w}{dx}\right) \qquad \ldots\ldots(3.12)$$

at x = 0 and x = ℓ — at the two ends of the body — since the temperature gradient there determines the heat flux into the skin. If this heat flux is finite since we are treating δ as an infinitesimal, it follows that

$$\frac{d\tau}{d\eta} = 0 \text{ at } \eta = 0 \text{ and } \infty. \qquad \ldots\ldots(3.13)$$

Provided that the skin is thin enough, the end effects will be unimportant; which is just as well, since there are many circumstances which could affect the precise nature of the end conditions. In other words, the skin temperature will be a maximum at the nose, and will be asymptotic to the zero-conductivity value at large distances from the nose. A numerical solution of (3.11) has been constructed to satisfy (3.13) and is shown in Figure 1. It will be seen that conductivity has little effect even as near the nose as where $\eta = 3$, i.e. $x = 3\delta^{8/13} L$, and that the maximum value of τ is

$$(\tau)_{\eta=0} = 1.11.$$

Hence, the maximum skin temperature is determined as

$$T_{max} = 1.11\, T_m \left[\frac{2\, T_m\, k\, d}{L^2 (k_H)_m \rho_1 U_1^3}\right]^{-1/13}. \qquad \ldots\ldots(3.14)$$

After some rearrangement, from the definition of T_m, it follows that

$$T_{max} = 1.11 \left[\frac{Q^4}{k\, d\, s^3 (\varepsilon_W + \varepsilon_B)^3}\right]^{1/13} \qquad \ldots\ldots(3.15)$$

if

$$Q = k_H \tfrac{1}{2} \rho_1 U_1^3 \sqrt{x},$$

where, by (3.10), Q is - by assumption - independent of x, and where the other parameters are

k = thermal conductivity of skin

d = thickness of skin

s = Stefan's constant

ε_W = emissivity of body outer surface

and ε_B = net rate of heat emission from body inner surface $\div\ s T_B^4$, (T_B being the temperature of the inner surface $= T_W$).

Equation (3.14) shows that although the quantity in square brackets (which we have called δ) is assumed $\ll 1$, it occurs raised to such a small negative power that it follows that T_{max} is often not very much larger than the representative body temperature T_m. To take a typical case where δ might be of the order of 1/1000, then $\delta^{-1/13}$ is only 1.7 and $T_{max} = 1.87\, T_m$. Of course the concept of a representative temperature was conceived chiefly for the purpose of introducing a non-dimensional notation, equation (3.15) shows that T_{max} is independent of this particular temperature and of the representative length L, as we would expect. It is affected only by downstream conditions insofar as we have assumed in (3.10) a simplified 'mean' value of T_W in stating the value of k_H. In fact, since conductivity is extremely local in its effects, it would seem justifiable to base k_H on the local body temperature T_{max}, although there is no analytical justification for this step within the accuracy of our estimates here. On the other hand, of course, if (as is assumed) T_{max} is not of the same order as T_{th}, then Q will in (3.15) be virtually constant over the entire surface, and the choice of an appropriate value for T_W in k_H is not important numerically.

By the same token, in an expression such as (3.10) we have not included the effects of a transition to turbulence which would relatively increase the heat transfer at stations behind the transition point; but provided the extent of laminar flow is such as to allow us to assume that transition occurs at a large value of η, the analysis need not take account of its presence, and we may neglect the effect of transition upon conditions near the nose - treating the flow as laminar everywhere.

4. Numerical Calculation

Bearing in mind the remarks at the end of the last paragraph, we shall in (3.15) use a value of k_H valid for the laminar boundary layer as given by (2.2) and (2.3), replacing T_W in these expressions by T_{max}. That is

$$Q = 0.15\,\rho_1 U_1^3 \sqrt{\frac{\nu_1}{U_1}} \left\{ \frac{\left[1 + \tfrac{1}{2}\sigma^{\frac{1}{2}}(\gamma-1)M^2 - \frac{T_{max}}{T_1}\right]\left[1 + \frac{11}{9}\frac{T_{max}}{T_1} + \frac{1}{5}\sigma^{\frac{1}{2}}(\gamma-1)M^2\right]^{-(1-\omega)/2}}{\sigma^{2/3}(\gamma-1)M^2} \right\} \quad \ldots\ldots (4.1)$$

Then, putting this value into (3.15) we find that

$$\frac{T_{max}}{T_1} = 0.96 \left(\frac{\rho_o^2 a_o^{10} \mu_o^2}{k\,d\,s^3\,T_o^{13}} \right)^{1/13} \left(\frac{\rho_1}{\rho_o}\right)^{2/13} \left(\frac{T_1}{T_o}\right)^{\left(\frac{2\omega-8}{13}\right)} \left[\frac{M^2}{\sigma^{8/3}(\gamma-1)^4}\right]^{1/13} \times$$

$$\times \left\{ \left[1 + \frac{\sigma^{\frac{1}{2}}(\gamma-1)}{2} M^2 - \frac{T_{max}}{T_1}\right]\left[1 + \frac{11}{9}\frac{T_{max}}{T_1} + \frac{\sigma^{\frac{1}{2}}(\gamma-1)}{5} M^2\right]^{\left(\frac{\omega-1}{2}\right)} \right\}^{4/13} \quad \ldots\ldots (4.2)$$

which is an equation connecting $\left(\frac{T_{max}}{T_1}\right)$ with the known parameters, where suffix 0 denotes characteristics of the air at some reference condition which we shall take to be at $0^{o}C$ at 1 atm. pressure, so that

$$\frac{\rho_o^{2} a_o^{10} \mu_o^{2}}{k\, d\, s^{3}\; T_o^{13}} = 2.43 \times 10^{7}$$

if we choose k = 0.1 cals./cm. sec. deg. C, and d = 1 mm. as representative values (corresponding to, say, 19 gauge steel). For other values of k and d, T_{max} varies inversely as their thirteenth root. We shall also take

$\gamma = 1.40$, $\sigma = 0.72$, $\omega = 0.60$, and $\varepsilon_W + \varepsilon_B = 1$

so that for a flat plate at zero incidence to the main stream

$$\frac{T_{max}}{T_a} = 5.0\left(\frac{\rho_a}{\rho_o}\right)^{2/13}\left(\frac{T_o}{T_a}\right)^{0.523}\left(1+0.17M^2-\frac{T_{max}}{T_a}\right)^{4/13}\left(1+0.07M^2+1.2\,\frac{T_{max}}{T_a}\right)^{-4/65} M^{2/13} \qquad \ldots\ldots(4.3)$$

where T_a is the temperature of the ambient air.

In Figure 2 the value of T_{max}/T_a is shown plotted against M for various altitudes of operation. The formula breaks down for low M because the value of T_{max} is there very near to T_{th}, and this invalidates the arguments used in this discussion; it is also then invalid by reason of the neglect of atmospheric radiation which, as (T_{max}/T_a) nears unity, becomes all-important. Quoted in this form, the variation of the ambient air temperature T_a with altitude is required before we may estimate T_{max}, and the variation assumed in these calculations is given in Figure 3.

We have here also assumed that conditions outside the boundary layer correspond to those of the ambient air; this is not necessary as our discussion is applicable to bodies without a pressure gradient, and not necessarily to flat plates at zero incidence. Assuming an adiabatic variation of state between ambient air conditions (denoted by subscript a) and those just outside the boundary layer, we find that the maximum temperature is increased or decreased in the approximate ratio:

$$\frac{T_{max}}{(T_{max})_{\text{flat plate}}} = \left(\frac{p_1}{p_a}\right)^{1/4} . \qquad \ldots\ldots(4.4)$$

For 'slender' shapes, this is a relatively unimportant correction.

It is important to notice that the extent of the region of high temperature, different from the zero-conductivity temperature, is very small. We have, in fact, that

$$x = \eta\, \delta^{8/13}\, L$$

and on substituting for δ

$$x = \eta \left[\frac{k^4 d^4}{s(\varepsilon_W + \varepsilon_B) Q^3} \right]^{2/13} = 1.17\eta \sqrt{\frac{kd}{s(\varepsilon_W + \varepsilon_B) T_{max}^3}} \qquad \ldots\ldots(4.5)$$

Hence, with the values of k and d selected as before, the position $\eta = 1$ corresponds to

$$x = \frac{10^5}{T_{max}^{3/2}} \text{ cm.} \quad (T_{max} \text{ in deg. K}). \qquad \ldots\ldots(4.6)$$

If T_{max} is as high as 1000° K, the temperature would have dropped to 900° K (at $\eta = 1$) just over an inch back from the nose; for the position $\eta = 1$ corresponds - as will be seen from Figure 1 - to a temperature equal to about 90 per cent of the maximum.

5. Experimental Evidence

Little evidence can be advanced to support these estimates of maximum temperature, simply because the missiles which reach the speeds and altitudes for which the correction due to conductivity is substantial are few and far between. The V.2 is an obvious example, but American tests give only the temperature at stations a foot or more behind the nose and not a few inches behind, where the maximum temperature is reached. However, an examination of the fins and body of operational V.2 rockets, after impact, by metallurgists suggested that a temperature of at least 650°C was reached, and using the data of this note we would calculate a maximum temperature of some 700°C during descent (at about 60,000 ft. altitude). Bearing in mind that, from (4.5), this would have dropped by over 100°C within an inch from the nose, all that can be said is that the order of calculated answer is probably correct in this case. The highest thermometer temperature reached in the trajectory corresponded to 1150°C.

On the other hand, calculations assuming the skin to be a non-conductor (but of finite thermal capacity) have shown good agreement with experimental results at the measuring positions, which is precisely what our theory would indicate - that, except within an inch or so of the nose, conduction of heat along the surface is unimportant.

6. Conclusions

(i) The effects of heat conduction, within the thin insulated skin of a two-dimensional plane-surfaced body, on the temperature distribution, arising from the heat transfer from the homogeneous boundary layer over the body in steady flight, have been examined: it is found that such effects may be accounted for approximately.

(ii) The effects of heat conduction are only important in the immediate neighbourhood of the nose of the body - in a typical case, within the first inch or two of the skin - and here heat is conducted back along the surface reducing the temperature near the nose and increasing it further aft, compared with that temperature which would be reached on a non-conducting skin. The temperature distribution is shown in Figure 1 plotted against a non-dimensional length η, which is evaluated in equation (4.5).

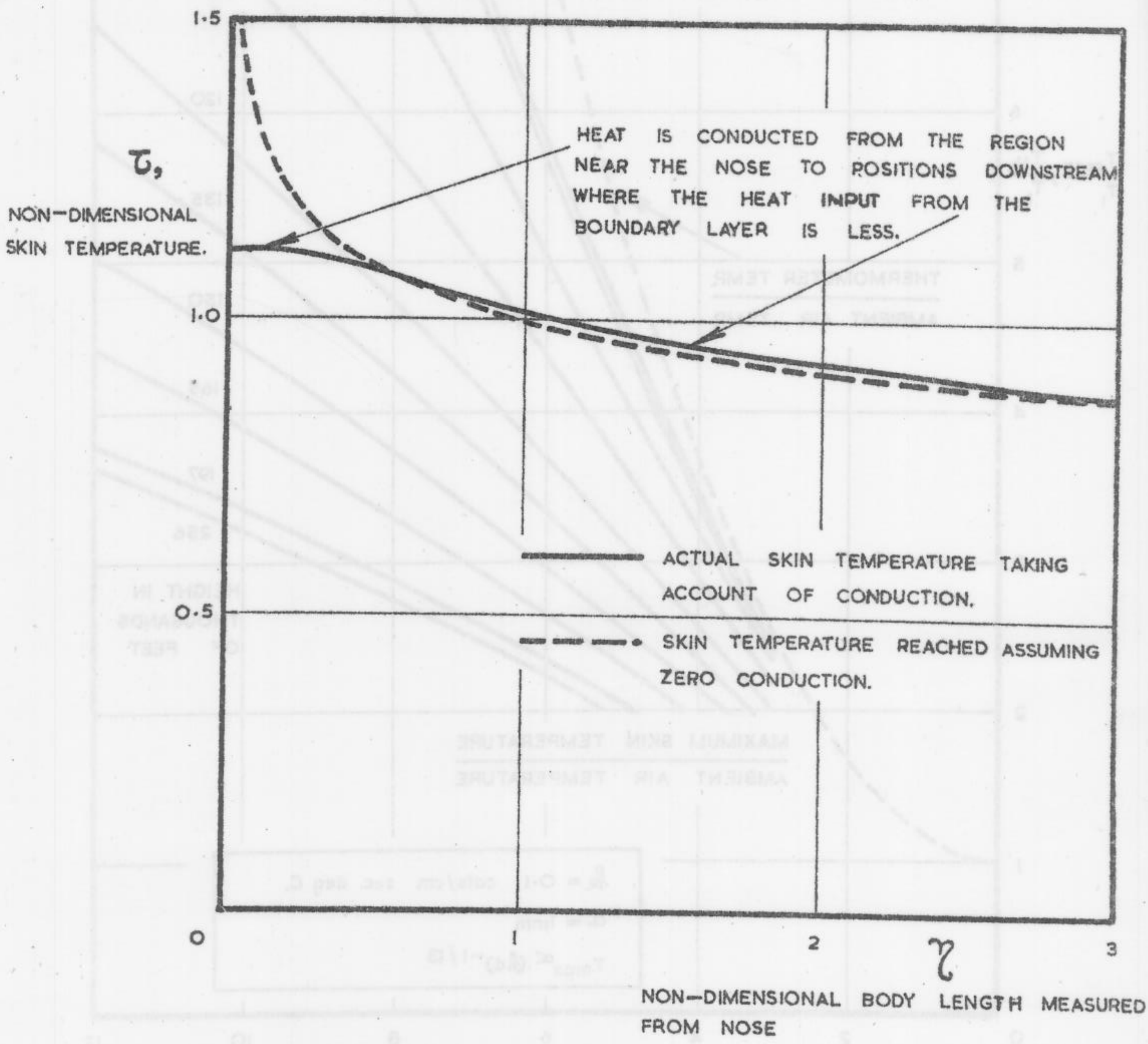
(iii) This modification to the temperature distribution causes the maximum temperature of a conducting surface to be reached at the nose, but to be less than the theoretical 'thermometer temperature', by an amount which is appreciable at high Mach Numbers or high speeds of flight. This temperature is given by equation (3.15), and is evaluated in a typical case for a range of flight conditions in Figure 2.

(iv) Elsewhere than near the nose, it is concluded that the skin acts as a non-conductor, so that an analysis ignoring the longitudinal and transverse temperature gradients existing in the skin, but calculating the rate of increase of temperature by relating the heat input to the thermal capacity of the body, should (over most of the skin) give an adequately approximate answer.

(v) The effect of the molecular structure of the air may impose a limit on the rate of heat transfer, and so on the skin temperature, which might invalidate the assessment here of maximum temperature.

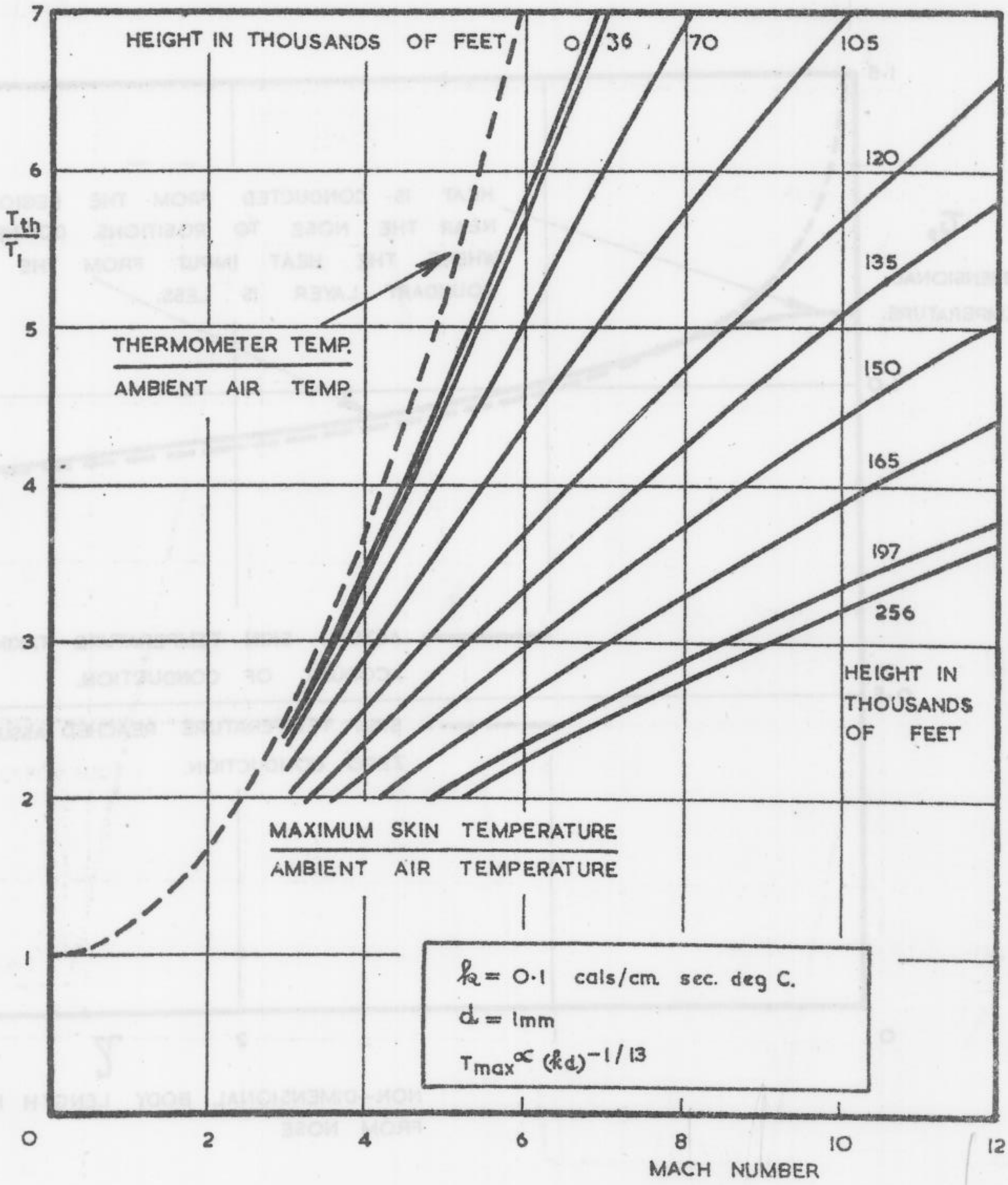
- - - - - -

FIG. 1.



THE EFFECT OF THERMAL CONDUCTIVITY ON THE TEMPERATURE DISTRIBUTION IN A THIN PLANE SKIN.

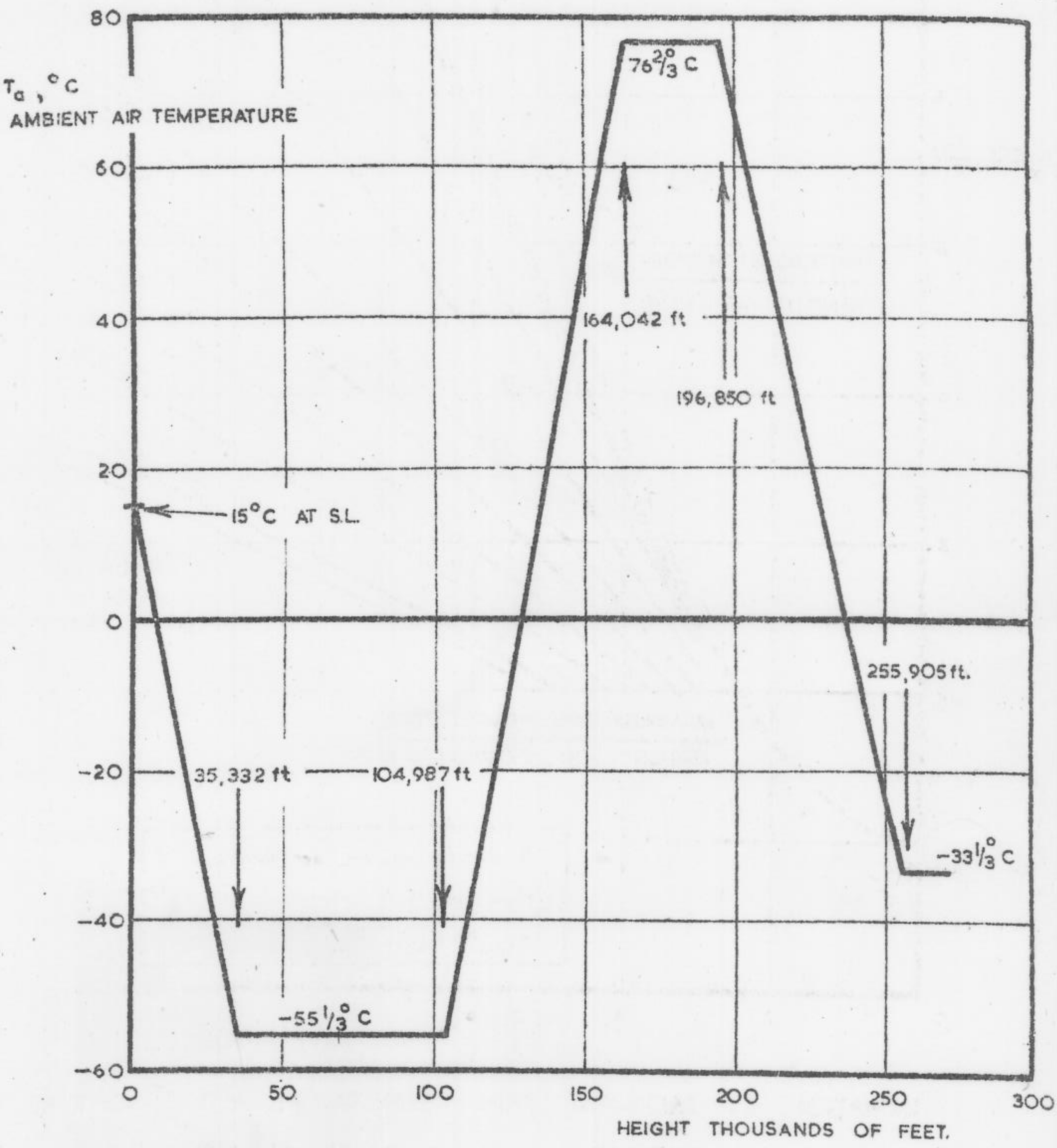
FIG. 2.



VARIATION OF MAXIMUM SKIN TEMPERATURE WITH FLIGHT MACH NUMBER & HEIGHT, FOR A FLAT PLATE BEHAVING AS A PERFECT BLACK BODY EMITTER ON THE OUTSIDE & INSULATED ON IT'S INNER SURFACE.

COLLEGE OF AERONAUTICS
REPORT No. 59.

FIG. 3.



ASSUMED DISTRIBUTION OF AIR TEMPERATURE WITH HEIGHT

(N.A.C.A. STANDARD)

104987
35332
140319
70,150

REFERENCES

No.	Author	Title, etc.
1.	Crocco, L.	Lo Strato Limite Laminare nei gas (Laminar Boundary Layer in Gas). Monografie Scientifiche di Aeronautica No.3, 1946 (R.A.E. Translation 218).
2.	Young, A.D.	Skin Friction in Laminar Boundary Layer in Compressible Flow. Aero. Quart. Vol.I, Part II, August 1949, pp.137-164.

R 7792
7186